INDETERMINACY AND INDETERMINISM.

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I.—*By* C. D. Broad.

In any discussion between professional physicists and professional philosophers there is a danger that the technical terms used by one party may be misunderstood by the other. I propose therefore to begin by explaining what I mean by "indeterminism," and at a later stage to state what I understand by "the principle of indeterminacy in physics." On the first point I shall expect Prof. Eddington to accept my definition of what is, after all, a technical term in my own subject. On the second point I am quite likely to be mistaken, and I shall welcome and profit by correction from Prof. Eddington.

(1) Preliminary Definitions.

It is best to begin by defining "determinism," and then to define "indeterminism" as the denial of determinism. But, before doing this, it will be convenient to define certain other notions which are involved in any possible statement of either doctrine.

I begin with the notion of a substance S *continuing to have* a certain characteristic ψ, at a moment t. I am going to use the phrase "the moment t *falls within* the period τ" to mean that t is neither the first nor the last moment of τ but is some moment between these two. The definition can now be given. It runs as follows:—

(1, 1) "S continues to have ψ at t" means "There is a

period τ, such that t falls within τ and S has ψ at every moment within τ."

I must next define the statement that S *changes* with respect to ψ at t. The definition runs as follows :—

(1, 2) "S changes in respect of ψ at t" means "S has ψ at t; and there is no period τ, however short, such that t falls within τ and S has ψ at every moment within τ."

It may of course happen that S continues to have a certain determinable characteristic Ψ at t, and at the same moment changes in respect of a certain determinate value ψ of this determinable. Thus, *e.g.*, S may continue to be coloured at t, but may change at that moment in respect of being red. Or it may continue to be red at t, and may change at that moment in respect of being scarlet.

Now it follows at once from these definitions by formal logic that, for any substance S, any characteristic ψ, and any moment t, there are three and only three mutually exclusive and collectively exhaustive alternatives, *viz.*, (i) that S has not ψ at t, (ii) that S continues to have ψ at t, and (iii) that S changes with respect to ψ at t. It is logically necessary that one or other of these alternatives should be fulfilled, and it is logically impossible that more than one of them should be fulfilled. I propose to refer to these three alternatives as "the logically possible states of S with respect to ψ at t." We are now in a position to define "determinism." The definition is as follows :—

(1, 3) "Determinism" is the name given to the following doctrine. Let S be any substance, ψ any characteristic, and t any moment. Suppose that S is in fact in the state σ with respect to ψ at t. Then the compound supposition that everything else in the world should have been exactly as it in fact was *and* that S should instead have been in one of the other two alternative states with respect to ψ is an impossible one.

I will make two elucidatory remarks on this definition before going further. (i) The determinist need not, and generally does not, assert either the necessity or the impossibility of the *separate items* of this compound supposition. He does not say that it is necessary that everything else in the world up to the moment t should have been exactly as it in fact was. He does not say that it is impossible that S should have been in a different state with respect to ψ at t from the state in which it actually was at that moment. What he says is that the *combination* of sameness in the rest of the world up to this moment with difference in the state of S with respect to ψ at this moment is impossible. Each item in the conjunction, taken by itself, is neither asserted to be necessary nor to be impossible. But the conjunction of the two is declared to be impossible. (ii) In this connection it is worth while to remark that, if McTaggart's *Principle of Extrinsic Determination* were true, the supposition that S might have been in a different state from that in which it actually was, would *in itself* be impossible. For, according to this principle, any substance that had been in a different state at t from that in which S was at t would *necessarily* have been a different substance from S. Thus, on McTaggart's view, the conjunction which the ordinary determinist asserts to be impossible would be impossible because one item of it is so taken separately. This must be contrasted with the ordinary determinist position that the impossibility of the conjunction as a whole is due, not to the impossibility of either of its items taken separately, but to an incompatibility between them which makes their combination impossible.

We are now in a position to define "indeterminism." Since it is simply the denial of determinism, its definition will be as follows :—

(1, 4) "Indeterminism" is the name given to the following

doctrine. There is at least one substance S, one characteristic ψ, and one moment t, such that, although S was in fact in the state σ with respect to ψ at t, yet the compound supposition that everything else in the world should have been exactly as it in fact was up to this moment *and* that S should instead have been in one of the other two alternative states with respect to ψ at that moment is a possible one.

It has commonly been held by indeterminists that rational beings are the only substances which are known to be exceptions to determinism, and that the characteristic of voluntarily deciding on a certain alternative is the only one in respect to which they are known to be exceptional. Thus, in practice, the name "indeterminism" has generally been given to the doctrine that the voluntary decisions of rational beings might have been other than they in fact were even though everything else in the world up to the moment of decision had been exactly as it in fact was. I propose to call this particular application of indeterminism "volitional indeterminism." If this more specific doctrine be true, indeterminism, as defined by me, must be true. On the other hand, indeterminism, as defined by me, might be true even if volitional indeterminism were false. It might be that the voluntary decisions of rational beings *are* completely determined, and yet that some substances (*e.g.*, electrons) are not completely determined in respect of some of their states (*e.g.*, their motions from one orbit to another). Or, again, it might be that rational beings *are not* completely determined in respect of their voluntary decisions, but that indeterminism applies to other substances also (*e.g.*, electrons) or to rational beings in respect of other states beside voluntary decision (*e.g.*, the occurrence of mental images).

Before leaving the subject of definitions I must point out one distinction, which is really quite clear, but which, I cannot help thinking, has sometimes been ignored by writers on the

present topic. The distinction is this. It is one question whether voluntary decisions are or are not completely determined. It is an entirely different question whether they do or do not themselves determine effects. I have explained what the first question means; and it will be seen from my definitions that it has *no* reference to any moment *later than* that at which the decision takes place. The second question refers *wholly* to what happens *after* the occurrence of the decision. It is therefore plain that they must be different questions, though of course it does not follow from this that the answer which we give to one of them may not have some important indirect bearing on the answer which it is reasonable for us to give to the other. I will now define the proposition that a state of a substance is *causally ineffective*. The definition is as follows:—

(1, 5) "The state σ of a substance S with respect to a characteristic ψ at a moment t is causally ineffective" means "No subsequent state of any substance would have been other than it in fact was even though S had been in one of the two other logically possible states with respect to ψ at t."

It is plain from our definitions that it is logically possible that a volition should be completely determined and effective, or completely determined and ineffective, or incompletely determined and effective, or incompletely determined and ineffective.

(2) Arguments Independent of the Principle of Indeterminacy.

I have now completed the task of defining and elucidating the doctrines of determinism and indeterminism. The next stage of my discussion is the following. "What reasons were alleged for and against determinism and indeterminism *before* the introduction of the principle of indeterminacy in physics? And were they valid?" This naturally divides into two

questions, one about determinism and the other about indeterminism; and I will consider them in that order.

(2, 1) *Arguments for Determinism.*—There are two types of argument for determinism, the *a priori* and the empirical. The former is much the more important of the two, and I will take it first.

(2, 11) *The A Priori Argument.*—The *a priori* argument is simply that, on careful inspection, the proposition which I have given as the meaning of "determinism" seems self-evidently true, and the opposite of it seems self-evidently false. I believe that this is the real reason which most determinists have for their belief, even when they profess to base it on empirical grounds.

Now in all cases of alleged self-evidence it is important to guard against a certain source of error which I will explain and illustrate. Sometimes we think we see that a certain proposition q is *intrinsically* necessary when what we really see is not this but that q is a *necessary consequence* of another proposition p which we are tacitly assuming. When p is made explicit we may see that it is not itself necessary, and that, although q is a necessary consequence of it, q is not intrinsically necessary. The following would be an example. It seems to all philosophers in Oxford, and to many lesser men, to be intrinsically necessary that the circumference of any circle, no matter of what size, should bear the same ratio to its diameter. The fact is, however, that this is only a necessary consequence of certain of Euclid's axioms, including his axiom of parallels. Euclid's axioms are tacitly assumed. Yet his axiom of parallels, when explicitly stated and carefully inspected, is not found to be intrinsically necessary by competent mathematicians. When this is realized one becomes much less certain of the intrinsic necessity of the proposition about circles.

Now I am much inclined to suspect that the alleged self-

evidence of determinism is in precisely this position. It is and can be seen to be a necessary consequence of a certain view about the nature of the universe, which is tacitly assumed. But, when this view is explicitly stated and carefully inspected, it does not seem to me to be intrinsically necessary. And, when these facts are realized, the apparent necessity of determinism seems to fade away. If you ask a man why it seems to him impossible to suppose that S could have been in a different state at t from that in which it in fact was, whilst everything else in the world up to that date had been exactly as it in fact was, I am pretty sure that his answer will be as follows. He will say: "If everything else in the world up to that date had been exactly as it in fact was, and yet S had then been in a different state from that in which it actually was, S would have had to have a different inner nature from that which it in fact had. But anything that had had a different inner nature from S would not have been S but a substance of a different kind and therefore a different substance."

The tacit assumption now begins to emerge. The world is conceived as composed of a number of substances. In each substance a distinction is assumed to exist between an inner nature, variable states, and variable relations to other substances. It is assumed that every state of any substance at any moment is determined jointly by the inner natures and previous states of itself and other substances and by the previous relations of itself to other substances. And it is assumed that any substance which had had a different inner nature from a given substance would necessarily have been a substance of a different kind and therefore a different substance.

Plainly our next business is to clear up this notion of the inner nature of a substance. The cash-value of this conception is as follows. On previous occasions either this substance itself, or others which are assumed to be of the same kind as this,

have been placed in various determinate forms of the same determinable conditions C. By "conditions" I mean their states at the time, their relations to other substances at the time, and the states of these other substances at the time. On all these occasions the immediately subsequent state of this substance, or of these supposedly similar substances, was found to be some determinate form of a certain determinable E. The corresponding determinate values of C and of E were all found to fall under a certain general formula ϕ. It is assumed that this formula is characteristic of this substance; that if at any time this substance were put into conditions which were *any* determinate form of C its immediately subsequent state would be some determinate form of E; and that the determinate value of E would be that which logically follows from inserting the determinate value of C in the general formula ϕ. To ascribe a *property* ϕ to a substance just is to assert that its states are connected with its previous states and relations by such a formula. And to say that its states are *determined* by its previous states and relations and its inner nature is to say that the determinate values of the characteristics of its present states follow logically from inserting the determinate values of the characteristics of its previous states and of its relations in those formulæ which are its properties.

It is now necessary to qualify the above rough preliminary statement in order to make it accurate. It is acknowledged that some at least of the properties of a substance may change in course of time without the substance thereby ceasing to exist and being replaced by another substance. At certain periods in the history of a bit of iron it will have the magnetic property, and at others it will not. Similarly the same mass of water will sometimes have the properties of a solid, sometimes the very different properties of a liquid, and sometimes the very different properties of a gas. The view of the world which we

are trying to formulate deals with such cases by drawing a distinction between properties of various orders. Properties of the first order are causal formulæ in which none of the variables are themselves properties. A property of the second order is a causal formula in which at least one of the variables is a property of the first order and the remaining variables are not properties. Thus liquidity and gaseousness are first-order properties of water, but it is a second-order property of water to lose the property of liquidity and acquire that of gaseousness at a certain temperature under any given pressure and at a certain pressure under any given temperature. There could of course be properties of the third and higher orders, and the general way of defining a property of any assigned order is now obvious.

A property may be called *ultimate* when it is not itself a variable in any other property. It is plain that if a substance were to change in respect of an ultimate property this change would necessarily be undetermined. For to say that the change was determined would be to say that it followed logically from substituting certain determinate values for the remaining variables in a formula which involves this property as a variable. And to say that this property is ultimate is to deny that there is any such formula. The view which we are trying to expound assumes that no substance could change in respect of any of its *ultimate* properties. Such a change would be, not a *change in* a substance, but the *destruction of* a substance followed perhaps by the *generation of* another substance of a different kind. It is also assumed, and this is quite a different point, that if S in fact has any ultimate property σ then anything which had not had σ would necessarily have been a different substance from S.

We are now in a position to formulate what I will call the *Deterministic Assumption*. It consists of the following propositions.

(i) Every substance has a set of ultimate properties, each of which is of finite order.

(ii) No substance can change in respect of any of its ultimate properties.

(iii) Any substance whose ultimate properties had differed in any respect from those which S in fact has would necessarily have been a different substance from S.

(iv) The value of any variable property of a substance at any moment is inferrible from one or more of its higher-order properties by substituting in the latter the determinate values of its states and relations immediately before that moment.

(v) The state of a substance at any moment with respect to any characteristic which is not a property is inferrible from its first-order properties at that moment by inserting in one or more of them the determinate values of its states and relations immediately before that moment.

Determinism, as we have defined it, follows at once from the Deterministic Assumption. If the state of S at t with respect to any characteristic ψ had been different from what it was, whilst everything else had been as it was, it follows from (v) that one of the first-order properties of S would have had to be different from what it then was. Now this first-order property is either ultimate or not. If it is, S would have had a different ultimate property, and therefore by (iii) would have been a different substance. If it is not, then it follows from (iv) that some higher-order property would have had to be different, and therefore from (i) and (iv) that some ultimate property would have had to be different. Therefore by (iii) S would have had to be a different substance in this case too. So the Deterministic Assumption does entail the impossibility of the supposition which determinism asserts to be impossible.

Are all five propositions of the Deterministic Assumption

self-evident? It seems to me that (i), (ii) and (iii) are involved in the notion of substance. To be a substance *means* to be something with properties in the sense defined, and to continue to be the same substance involves, as part of its meaning, continuing to have the same fundamental modes of behaviour. Also I can attach no meaning to the supposition that the very same substance which in fact had certain fundamental modes of behaviour might instead have had different ones. I have nothing to say against these three propositions except the following. It is certain that, *if* there be substances, they must answer to these conditions. But whether it is certain that there are substances, and whether it is self-evident that all events must be states of substances, is another question.

Apart from this it is propositions (iv) and (v) which seem questionable. It does not seem to me self-evident that a substance must have so many first-order properties, and these of such a kind, that its states at every moment with respect to every characteristic down to the last degree of determinateness could be inferred by inserting the determinate values of its immediately preceding states and relations. It is surely possible, *e.g.*, that its first-order properties might suffice only to fix its state with regard to every determinable characteristic and to confine the determinate value of each of these within certain narrow limits. The precise values within these limits might not be inferrible from any or all of the first-order properties even when completely determinate values of all the immediately preceding states and relations were substituted for the variables in these properties. Similar remarks apply to proposition (iv). It is not self-evident that every variation in any property of a substance must be inferrible from some higher-order property by substituting for the variables in the latter the determinate values of the states and relations of the substance just before the change took place. The higher-order properties of

substance might quite well not be so numerous, or not be of such a form, that inferences of this kind would be theoretically possible in every case.

To sum up. Many people start by professing to find determinism self-evident. But, when one begins to enquire further, one soon finds that what is self-evident is that it follows from a certain general view, *viz.*, the Deterministic Assumption, which is tacitly presupposed. When this is explicitly formulated one part of it turns out to be merely an analysis of the notion of substance. This leaves it an open question whether there are substances, and whether every event must be a state of some substance. The other part turns out not to be self-evident even if there be substances and even if every event must be a state of some substance.

(2, 12) *Empirical Arguments.*—If determinism had proved to be either self-evident or a necessary consequence of self-evident premises, there would have been no need of empirical arguments for it, and no force in empirical arguments against it. As it is, however, such arguments become of some importance. Now the main empirical support for determinism was drawn from the material world. It was said that, whilst superficial appearances are partly for it and partly against it, the more accurately inanimate matter is studied the smaller becomes the area within which there is even an appearance of indeterminism. Eventually a position seemed to have been reached where it was obviously reasonable to ascribe the few remaining appearances of indeterminism in inanimate matter to our temporary ignorance of the laws of the configurations involved. There remained human minds, and those changes in human bodies which seem to be in part determined by events in the minds which animate such bodies. Admittedly there is a strong appearance of indeterminism in certain events which take place in human minds, particularly in some of their voluntary

decisions. On this topic determinists used the following arguments, some of which are strong and others weak.

(i) That there is good reason to believe that not all mental events are open to introspection. Therefore, even if all mental events were completely determined, and all the cause-factors in their total causes were themselves mental, some of these factors might be incapable of being detected by introspection.

(ii) That some of the cause-factors in the total cause of a mental event may be not mental but material. Any such factors would, *eo ipso*, be incapable of being detected by introspection. These two arguments are quite successful attempts to show that the appearance of indeterminism in the human mind is compatible with determinism, and might be expected even if determinism were in fact true. But they do not constitute any positive argument for mental determinism.

(iii) The alleged fact that determinism almost certainly holds for the inorganic world and that, in so many departments of the latter, apparent indeterminism has been found to be *only* apparent, suggests by analogy that determinism is probably true of minds and mental events in spite of appearances to the contrary. This, taken by itself, does not strike me as a particularly strong argument. Minds, mental events, and psychical causation are, on any view that is worth a moment's consideration, so extremely unlike matter, material events, and physical causation that an argument by analogy from determinism in the latter to determinism in the former, against all appearances to the contrary, is not very convincing.

(iv) The following argument is more to the point. It is asserted that all the motions of human bodies, including those which are commonly ascribed to volition, can be shown to be completely accounted for by physical causes. If this be so, volitions, whether determined or not, are causally ineffective. Yet it is plainly true that a volition generally is followed by

the movement that has been willed, and that certain series of bodily movements (*e.g.*, those involved in writing this paper) are never known to happen unless preceded by a corresponding volition. How is this almost invariable correlation between events which have no direct causal connection to be explained? The most plausible explanation seems to be that certain of the physical factors in the total cause of such bodily movements have mental as well as physical effects. The bodily movement would not have taken place unless a certain kind of physical state had preceded it, and this kind of physical state cannot take place without being accompanied or immediately followed by a desire for this bodily movement. Now this physical state, like all physical states, is completely determined. And, since all volitions are the necessary accompaniments or sequents of such physical states, all volitions must be completely determined.

This argument, though not without a certain superficial plausibility, contains several fallacies. (*a*) The utmost that the experimental facts warrant us in asserting is that all the physical energy involved in a voluntary movement is of purely physical origin. Volition neither adds to nor subtracts from the total energy in the physical world, including the agent's body and the materials stored up in it. This, however, does not prove that volition is causally ineffective. For the occurrence of a volition in my mind at a certain moment may be a necessary condition for some of the energy stored up in my body to change at that moment from the potential chemical form to the kinetic form of bodily movement. (*b*) The second premise of the argument is incompatible with our being able to know that the first is true. If certain kinds of bodily movement never have been found to happen without being preceded by corresponding volitions *as well as* by certain physical states, we cannot possibly *know* that the volitions are causally irrelevant, *i.e.*, that the same bodily movements would have

taken place if the physical pre-conditions had been the same but the volition had been absent. It is *possible* that volitions may be causally irrelevant to voluntary movements; but, in the absence of negative instances, our belief that they are so is an act of pure faith. And, by hypothesis, there are no negative instances.

These, I think, are the only important empirical arguments for determinism; and it is plain that they are quite inconclusive.

(2, 2) *Arguments for Indeterminism.*—I do not think that there has ever been an *a priori* argument for indeterminism, unless ethical arguments be counted as such. I propose to ignore ethical arguments as irrelevant to the topic set for discussion. Before the recent development of physics no one in modern times would have suggested that there is any empirical evidence for the occurrence of incompletely determined events anywhere but within the human mind. To the merely negative contention that for many of our decisions we cannot detect any completely adequate cause, and to the more positive contention that we have a conviction that such events are not completely determined, the determinist gives an answer which I have already mentioned and which seems to me to be satisfactory. But there is a much more serious empirical argument against mental determinism, which I will now state. The essence of this contention is that there are certain characteristic peculiarities of minds and of mental events which make the doctrine of mental determinism almost meaningless and quite incapable of even approximate empirical verification. The argument is as follows.

Minds and mental processes, even at their lower levels differ from inorganic matter and physical processes in two important respects. And, when we come to the higher levels of mind, such as reasoning and deliberation, there is a further

peculiarity to which there is absolutely no analogy even in organic matter.

(i) The first peculiarity is that physical causation is non-mnemic, whilst nearly all mental causation is at least ostensibly mnemic. If we want to account for the occurrence of a certain mental event at a certain moment it is hardly ever sufficient to refer to the immediately preceding introspectible states of the mind. In nearly every case we find that, whilst one factor of the total ostensible cause of a mental event is some other introspectible mental event which immediately precedes it, other equally necessary factors are mental events which took place in the remote past and may never have been thought of in the interval. If we take this ostensibly mnemic causation to be an ultimate fact, we have to admit that any event at any date in a mind's history may *directly* influence any subsequent state of that mind across the temporal gap between them. On such a view it is difficult to see what would be left of determinism, or what analogy there would be between physical and mental causation.

In practice, of course, no one does take this view. We always assume that causation which is ostensibly mnemic is really non-mnemic, *i.e.*, that all the cause-factors in the total cause of any event are temporally continuous with it. But this is made possible only by postulating something purely hypothetical and unobservable to fill the temporal gap between the remote factor of the ostensible cause and the effect. The past experience is assumed either to have started an imperceptible process π, which goes on during the interval, or to have made a structural modification μ, which persists through the interval. And the real total cause of the present mental event is supposed to be either the immediately preceding phase of the process π or the persistent modification μ, together with some immediately preceding stimulus or reminder.

At once we are faced with the following difficulty. When we make similar assumptions about matter we know fairly well what we are doing. We have seen minute structure and Brownian movements through microscopes, and we are merely postulating something still more minute of the same kind. We can therefore formulate our assumptions definitely; can draw inferences from them as to what we ought to observe under assignable conditions; and can then support, refute, or modify our hypothesis by the results of our observations. In psychology nothing of the kind is possible. If we suppose our μ's and π's to be purely mental, we must admit that we have no clear idea of mental "structure" and hardly a clearer idea of imperceptible mental process. If we suppose the μ's and π's to be physiological, we are not a whit better off. We know any such μ or π *only* under the description of "that physiological trace or process which is initiated by such and such an experience." And from this we can infer nothing as to its probable effects in the way of subsequent experiences under assigned conditions. Each μ or π is postulated entirely *ad hoc*, and we can work out no further observable consequences of the postulate, by which it might be supported, refuted, or modified. The result is that most causal "explanations" of determinate mental events by psychologists are the merest "eye-wash." They are on a level with the statements of Molière's physicians that opium makes one sleep because of its dormitive faculty.

(ii) The second peculiarity is the individuality of human minds, as contrasted with inorganic bodies, and their apparent lack of compositeness, as compared with both organic and inorganic bodies. There is a very limited number of kinds of matter, and every material system is composed of matter of one or more of these few kinds. Each kind of matter has its own characteristic chemical and physical constants, which

can be determined by experiments on *any* pure sample of the kind, and which persist unchanged throughout its history. There are certain general laws which apply to matter of *every* kind, *e.g.*, the laws of motion and gravitation. Once the laws are known and the constants have been determined, we can predict the behaviour of any purely material system by knowing the kinds of matter of which it is composed, the way in which they are arranged, and the circumstances in which it is to be placed. We have merely to substitute, in the general laws which apply to *all* matter, the particular constants for the particular kinds of matter with which we are concerned and the determinate values of the variable conditions.

Now there is hardly any analogy to this in human psychology. No doubt there are laws which apply to all minds, *e.g.*, the laws of retentiveness and of association. But, if we like to talk of "psychical constants," we must notice the following fundamental differences between them and physical constants. (*a*) Every different human mind has its own characteristic psychical constants. (*b*) Different minds cannot be regarded as composed of common materials of a few fundamental kinds combined in different proportions and arrangements. It follows that in order to discover the detailed causal laws which govern the mind of a certain man *Smith* you must investigate Smith himself. No amount of knowledge of how Jones's or Robinson's minds have worked will enable you to learn, either by direct inductive generalisation or by indirect deductive inference, the detailed laws of the workings of Smith's mind. (*c*) The psychical constants of a mind seem to be indefinitely numerous, and incapable of deduction from a few fundamental ones. So an exhaustive knowledge of them is not practically possible. (*d*) None of the psychical "constants" that we know of are really constant. They are liable to change, either gradually as we grow older, or suddenly as in cases of conversion, accident,

illness, lunacy, or falling in love. (*e*) These changes may of course be themselves determined. But to predict how a person's mode of behaviour under certain stimuli would change into a different mode of behaviour under the same stimuli would require a knowledge of psychical constants of the second order. Now these could not possibly be discovered until he had been converted or had fallen in love several times. Such incidents are not very common in the lives even of those who are most susceptible to religious or sexual emotion; and it is not at all certain that falling in love or finding salvation for the third time would have the same effects on one's first-order psychical constants as doing so for the first time.

Such facts as I have mentioned are perfectly compatible with the view that the possible present state of the mind is confined within fairly narrow limits by its past history and present circumstances. They still leave the notion of complete causal determination of all mental events down to the minutest detail *theoretically* possible. But they make it a purely theoretical ideal, which, since it is not self-evident, must be accepted, if at all, as a matter of mere faith. There is obviously no hope of even approximately verifying it empirically, or of making any detailed practical application of it.

(iii) I come now to a peculiarity which appears only at the higher levels of mind. I will take the case of what we call "coming to believe so-and-so on rational grounds." That many beliefs are not caused in this way is certain. It is also certain that we often think that a belief has been caused in this way when in fact it has been caused in some other way. But there is no doubt that sometimes an essential part of the cause of my beginning to believe q at a certain moment is the fact that I then see, or think I see, such a formal relation between q and another proposition p, which I already believe, that q must be true if p is so. And there is no reason to doubt that sometimes

I really do see such a formal relation and really am justified in extending my belief in p to q.

Now this is a type of causation to which there is nothing analogous either in matter or in the lower levels of mind. An essential factor in the cause is here the recognition by the mind at a certain moment of a timeless formal relation between two propositions. Thus a necessary condition of the occurrence of my belief in q is something which is neither an event nor a substance nor a property of a substance, but a timeless relation between two propositions. For, unless this timeless relation had held, I could not have intuited it at any time; and, unless I had intuited it when I did, I should not then have grounded a belief in q on my pre-existing belief in p.

This fact is, of course, quite compatible with the most rigid determinism of mental events. But it does show that, if the higher kinds of mental event be completely determined, the nature of the causation must be so unique that no analogies from inorganic matter, or organic matter, or the lower levels of mind can have any relevance to it. Consequently all such analogies are irrelevant to prove that the higher kinds of mental event *are* in fact rigidly determined, if once it be granted that they *may* not be so, that some of them *appear* not to be so, and that nothing but empirical evidence is admissible on the question.

I think it should now be plain that, quite apart from the principle of indeterminacy in physics, determinism is only a useful practical postulate. It is not axiomatic, and there neither is nor is ever likely to be anything approaching to adequate empirical evidence for it in psychology. It remains to formulate the principle of indeterminacy in physics, and to see what change, if any, it makes in the situation in which we already find ourselves with regard to determinism.

(3) STATEMENT OF THE PRINCIPLE OF INDETERMINACY.

I understand that there is an older and a newer form of this principle. The older form was current when the picture of an atom as consisting of a solar nucleus with planetary electrons rotating about it in a certain restricted set of physically possible orbits was taken seriously. On that view of the atom radiation from it takes place when and only when an electron jumps from one to another of the limited set of its physically possible orbits. And the principle of indeterminacy asserted that the jumping or not jumping of an electron from one possible orbit to another at a given moment was undetermined.

This pictorial representation of the atom is no longer taken seriously, and the more recent form of the principle of indeterminacy, as I understand it, may be stated as follows. There are certain measurable magnitudes, q and p, of fundamental importance in physics. The former is of the nature of position, the latter of the nature of momentum. Relatively to certain data, which I will denote by D_{pq}, the value of p at a certain time and place is equally likely to fall within or without a certain small range Δp. Relatively to the same date the value of q at the same time and place is equally likely to fall within or without a certain small range Δq. As the datum D_{pq} is varied continuously in certain respects the range Δp contracts without limit. As the datum D_{pq} is varied continuously in certain other respects the range Δq contracts without limit. But these two types of variation in the data are so interconnected that any variation which diminishes the range Δp increases the range Δq, and any which diminishes Δq increases Δp. And the interconnection is such that the product of Δp by Δq has a certain characteristic value which is independent of variations in the datum D_{pq}. It is impossible to find any datum relevant to determining the probable values of p and of q which does not lead to this result.

The above is not precisely the form in which I have seen the principle stated in books. All statements which I have seen talk of *the* probability of the value of a certain variable falling within certain limits, and make no mention of the datum with respect to which this probability is reckoned. This is nonsensical, if taken literally ; and I have avoided the appearance of such nonsense by explicitly mentioning a datum D_{pq}. Again, I find it stated that either p or q, taken separately, could be measured with complete accuracy. This again is plainly nonsense, if taken literally ; so I have substituted a statement about approximating indefinitely to a limit, in order to avoid the appearance of such nonsense. I do not of course accuse the eminent physicists who have formulated the principle either of believing nonsense or of being incapable of stating clearly what they have in mind. I take it that they are writing for a special set of readers who will supply for themselves the necessary interpretations and supplementations, and that they are quite justified in sacrificing pedantic accuracy for convenience and brevity. But, in order to attach any clear meaning to their statements myself or to convey anything intelligible to other non-experts, I find it necessary to put the principle in the way in which I have put it. I am quite aware that, in the process of reformulation, I may have distorted or unjustifiably added to the meaning of the physicists. If I have done so, Prof. Eddington will, no doubt, correct me.

(4) The Bearing of Indeterminacy on Indeterminism.

After all these preliminaries, I believe that the question which we were asked to discuss can be answered clearly in a very few lines. I will consider in turn the older and the newer form of the principle of indeterminacy.

(4, 1) *The Older Form of Indeterminacy.*—The older form of the principle of indeterminacy in physics is relevant to

the question of indeterminism in so far as it still further weakens the two empirical arguments for mental determinism which I have numbered (iii) and (iv) in Section (2, 12) of this paper. Each of those arguments assumes as a premise that all physical events are completely determined. The principle which we are at present considering denies this. We have already shown that these two arguments are extremely weak even if the premise of complete physical determinism be admitted. They are naturally weakened still more by anything which casts doubt on this premise.

(4, 2) *The Later Form of Indeterminacy.*—The experimental f cts which are summed up and generalised in the later form of the principle of indeterminacy seem to be susceptible of at least two explanations. (i) They may depend on the fact that when experiments are conducted to measure certain magnitudes beyond a certain degree of accuracy the effect of the measuring instrument on the process to be measured ceases to be negligeable. The instruments themselves are presumably made of the same ultimate stuff, obeying the same ultimate laws, as the objects and processes which they are used to measure. It might well be that, after a point, any device which reduced the disturbing effect of the instrument in so far as it measured p would inevitably increase its disturbing effect in so far as it measured q, and conversely. And it would not be surprising that the product $\Delta p \ \Delta q$ should have a certain characteristic value no matter how p and q were measured.

If the above were the right explanation, I do not see that the principle of indeterminacy would have any bearing whatever on the question of mental determinism or indeterminism. But it would be extremely interesting as showing that, after a certain point, physicists are faced with a difficulty analogous to one which has long been felt in psychology. The difficulty is that the ultimate data of psychology have to be discovered by

introspection, that introspection has to be performed by a mind on itself and its states, and that there is reason to fear that the process of introspection may interfere with the objects and processes to be introspected. Since this is one of the reasons alleged by behaviourists for deserting introspective psychology, as not scientifically respectable, and confining themselves to the study of bodily behaviour, the latest peccadillos of physics must have produced a painful impression in American Let us hope that the scandal has been kept from Prof. Watson.

(ii) The above interpretation of the principle of indeterminacy would make it epistemic, in a rather widened sense of that term, and not ontological. But it is quite possible to suggest an ontological interpretation of it. It is often forgotten that the notion of "the value of a certain determinable at a certain point or a certain instant" is always a highly artificial and sophisticated one. Consider, *e.g.*, the notion of the direction or the curvature of a curve at a point, and the notion of the velocity or the acceleration of a particle at an instant. If taken literally, all these notions are contradictions in terms. Often a clear meaning can be given to them; and, when this is so they are often useful and even indispensable notions. But whenever a meaning can be given it is always in terms of limits, and on the assumption that certain functions are continuous, that they are differentiable, that their differential coefficients are themselves finite, continuous, and differentiable, and so on. These are very special conditions indeed. They are not always fulfilled even in the case of quite commonplace curves. *E.g.*, at a singular point on a curve you may have to say that it has no curvature, or an infinite curvature, or two different curvatures, etc. There is not the least reason why all the determinables which are of fundamental importance in physics should be such that it is possible to give a meaning to the notion of the value of such a variable at a point or at an instant. Still less can we

be sure beforehand that they must be such that it is possible to give a meaning to the notion of the rate of change in the value of such a variable at a point or an instant. Now, if we were dealing with variables which were tacitly assumed to fulfil these conditions, but which in fact did not do so, we should be likely to be faced sooner or later with paradoxes which would be a sign of the discordance between our tacit assumption and the actual facts. It seems to me very likely that the facts which are summarised in the principle of indeterminacy are the indications of a false assumption of this kind.

Let me take a very crude analogy, which may throw some light on this interpretation. Let there be two surfaces S_1 and S_2 which intersect in a line l_{12}. Let the surface S_1 be red all over, and let the shade of red vary continuously towards and away from l_{12}. Let S_2 be green all over, and let the shade of green vary continuously towards and away from l_{12}. Then, if we confine our attention to S_1, we can assign a perfectly definite meaning to the notion of "the colour of the line l_{12}." It will be a certain perfectly determinate shade of red. Similarly, if we confine our attention to S_2, we can assign a perfectly definite meaning to the notion of "the colour of the line l_{12}." It will be a certain perfectly determinate shade of green. But, if we consider S_1 and S_2 together, we shall have to say either that l_{12} has at the same time two different colours, or that it has no colour at all. I do not suggest that there is complete analogy between this and the fact that p without q or q without p can be determined with unlimited accuracy, whilst any attempt to assign absolutely determinate values to both together fails. But there is enough analogy to make this example of some use in illustrating the second interpretation of the principle of indeterminacy.

With the above interpretation of the principle, its bearing on the question of determinism or indeterminism is as follows.

Our statement of the doctrine of determinism in Section (1) of this paper presupposed the notion of momentary states. This in turn presupposed that the characteristic ψ was always of such a kind that a clear meaning can be given to the statement that "S has ψ at t," where t is an instant and ψ is absolutely determinate. If this assumption breaks down for any characteristic, determinism, as defined by us, has no application to the state of a substance with respect to that characteristic. Now, if there be such characteristics in physics, it seems even more likely that some of the characteristics of mental events are of this kind. It is possible that the definition of "determinism" might be modified and extended to cover the case of such characteristics, but I must confess that I do not see clearly at present how this could be done.